



## Herhalingsoefeningen: ontbinding in factoren

### 1) Distributieve eigenschap:

a)  $a(x - y) + b(x - y) =$

b)  $9a^2b(x + y) - 3ab^2(x + y) =$

c)  $5a^2(b - 2) - 15a(2 - b) =$

d)  $(3x - 1) - 3x + 1 =$

e)  $5(x^2 + 7) - 14 - 2x^2 =$

f)  $ax + x =$

g)  $a^3b^4 - a^4b^3 =$

h)  $24a^3b^5 - 60ab^4 =$

i)  $(x - 2)y - z(2 - x) =$

j)  $x + y(x + 3) + 3 =$

k)  $x(x^2 + y) - x^2 - y =$

### 2) Ontbinding van tweetermen: verschil van even machten:

a)  $a^2 - 1 =$

b)  $25a^2 - 81 =$

c)  $16x^2 - 25y^4 =$

d)  $49a^2 - 100b^2 =$

e)  $5a^2 - 125 =$

f)  $ab^3 - a^3b =$

g)  $98x^2 - 2y^6 =$

h)  $x^4 - 16 =$

i)  $a^8 - 256 =$

j)  $a^{16} - b^{16} =$

k)  $x^5 - 16xy^4 =$

### 3) Ontbinding van drietermen: volkomen tweedemacht:

a)  $x^2 - 2xy + y^2 =$

b)  $16a^2 - 24ab + 9b^2 =$

c)  $x^4 + 6x^2 + 9 =$

d)  $\frac{1}{4}x^6 - \frac{2}{3}x^5 + \frac{4}{9}x^4 =$

e)  $x^2 - 2 + \frac{1}{x^2} =$

f)  $75a^6b^2c^2 + 108a^2b^8c^2 + 180a^4b^5c^2 =$

g)  $\frac{9}{4}x^4y - x^3y^2 + \frac{x^2y^3}{9} =$

h)  $(2a - 1)^2 - 4(2a - 1) + 4 =$

i)  $0,04a^2 + 0,04ab + 0,01b^2 =$

### 4) Ontbinden van viertermen: derdemacht van een tweeterm:

a)  $a^3 + 3a^2 + 3a + 1 =$

b)  $8x^3 + 12x^2y + 6xy^2 + y^3 =$

c)  $27a^3x^6 - 54a^2bx^4 + 36ab^2x^2 - 8b^3 =$

d)  $\frac{1}{27}a^6 - a^5 + 9a^4 - 27a^3 =$

### 5) Ontbinden van viertermen: termen groeperen, 2 en 2:

a)  $ax + bx + ay + by =$

b)  $a - ab + 3 - 3b =$

c)  $x^4 - 2x^3 + 4x - 8 =$

d)  $a^2y - b^2 + b^2y - a^2 =$

e)  $a^3 + a^2b + ab^2 + b^3 =$

f)  $6a^3 + a^2 - 18a - 3 =$

g)  $15x^2 - 30xy + 8xz - 16yz =$

h)  $(x + y)^2 + (x - y)(x + y) =$

i)  $(a - 2b)(a - 4b) - (a - 2b)^2 =$

6) Ontbinden van viertermen: termen groeperen, 3 en 1:

- a)  $x^2 + 2x + 1 - y^2 =$
- b)  $a^2 - c^2 + 4ab + 4b^2 =$
- c)  $16a^2 - 9b^2 - 4c^2 + 12bc =$
- d)  $x^4 - 9b^2x^2 - 6bc^2x - c^4 =$

- e)  $25a^2 - 20ab + 4b^2 - 9a^4 =$
- f)  $c^2 - d^2 - 2c + 1 =$
- g)  $4 - a^2 - 2ax - x^2 =$

7) Ontbinding in factoren: allerlei:

- a)  $a^2 - ab =$
- b)  $a^2 - 4y^4 + 12x^3y^2 - 9x^6 =$
- c)  $32x^4 - 2a^4 =$
- d)  $27x^3 - 27x^2y + 9xy^2 - y^3 =$
- e)  $36a^4b^3c - 24a^2b^2c + 4bc =$
- f)  $27a^5 + a^2 - 108a^3 - 4 =$
- g)  $25a^6b^2 - 10a^5b^3 =$
- h)  $125a^3 - 75\frac{a^2}{b} + 15\frac{a}{b^2} - \frac{1}{b^3} =$
- i)  $(a+b)^2 + 2(a+b) + 1 =$
- j)  $20a^3 + 5a^2 - 4a - 1 =$
- k)  $a^2 - 6a + 9 - b^2 =$

- l)  $\frac{1}{9}a^6b^4 - \frac{2}{9}a^5b^2 + \frac{1}{9}a^4 =$
- m)  $x(a-1) - y(1-a) =$
- n)  $a^2 - 2ab + 2bc - ac =$
- o)  $x^{16} - 1 =$
- p)  $8a^5 + 24a^3b^4 + 18ab^8 =$
- q)  $64x^6 - 1 =$
- r)  $150ab^2 + 8a^3 - 60a^2b - 125b^3 =$
- s)  $a^3 - b^4 + a^2b - ab^3 =$
- t)  $4x^4 - 81 =$
- u)  $3a^4b + 24ab^4 =$



## Herhalingsoefeningen: ontbinding in factoren, oplossingen

### 1) Distributieve eigenschap:

- a)  $a(x - y) + b(x - y) = (x - y)(a + b)$   
b)  $9a^2b(x + y) - 3ab^2(x + y) = (x + y)(9a^2b - 3ab^2) = 3ab(x + y)(3a - b)$   
c)  $5a^2(b - 2) - 15a(2 - b) = 5a^2(b - 2) + 15a(b - 2) = (b - 2)(5a^2 + 15a) = 5a(b - 2)(a + 3)$   
d)  $(3x - 1) - 3x + 1 = (3x - 1) - (3x - 1) = (3x - 1)(1 - 1) = 0$   
e)  $5(x^2 + 7) - 14 - 2x^2 = 5(x^2 + 7) - 2(7 + x^2) = (x^2 + 7)(5 - 2) = 3(x^2 + 7)$   
f)  $ax + x = x(a + 1)$   
g)  $a^3b^4 - a^4b^3 = a^3b^3(b - a)$   
h)  $24a^3b^5 - 60ab^4 = 12ab^4(2a^2b - 5)$   
i)  $(x - 2)y - z(2 - x) = (x - 2)y + z(x - 2) = (x - 2)(y + z)$   
j)  $x + y(x + 3) + 3 = y(x + 3) + (x + 3) = (x + 3)(y + 1)$   
k)  $x(x^2 + y) - x^2 - y = x(x^2 + y) - (x^2 + y) = (x^2 + y)(x - 1)$

### 2) Ontbinding van tweetermen: verschil van even machten:

- a)  $a^2 - 1 = (a + 1)(a - 1)$   
b)  $25a^2 - 81 = (5a + 9)(5a - 9)$   
c)  $16x^2 - 25y^4 = (4x + 5y^2)(4x - 5y^2)$   
d)  $49a^2 - 100b^2 = (7a + 10b)(7a - 10b)$   
e)  $5a^2 - 125 = 5(a^2 - 25) = 5(a + 5)(a - 5)$   
f)  $ab^3 - a^3b = ab(b^2 - a^2) = ab(b + a)(b - a)$   
g)  $98x^2 - 2y^6 = 2(49x^2 - y^6) = 2(7x + y^3)(7x - y^3)$   
h)  $x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x + 2)(x - 2)$   
i)  $a^8 - 256 = (a^4 + 16)(a^4 - 16) = (a^4 + 16)(a^2 + 4)(a^2 - 4)$   
 $= (a^4 + 16)(a^2 + 4)(a + 2)(a - 2)$   
j)  $a^{16} - b^{16} = (a^8 + b^8)(a^8 - b^8) = (a^8 + b^8)(a^4 + b^4)(a^4 - b^4)$   
 $= (a^8 + b^8)(a^4 + b^4)(a^2 + b^2)(a^2 - b^2)$   
 $= (a^8 + b^8)(a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$   
k)  $x^5 - 16xy^4 = x(x^4 - 16y^4) = x(x^2 + 4y^2)(x^2 - 4y^2) = x(x^2 + 4y^2)(x + 2y)(x - 2y)$

### 3) Ontbinding van drietermen: volkomen tweedemacht:

- a)  $x^2 - 2xy + y^2 = (x - y)^2$   
b)  $16a^2 - 24ab + 9b^2 = (4a - 3b)^2$   
c)  $x^4 + 6x^2 + 9 = (x^2 + 3)^2$   
d)  $\frac{1}{4}x^6 - \frac{2}{3}x^5 + \frac{4}{9}x^4 = \frac{1}{36}x^4(9x^2 - 24x + 16) = \frac{1}{36}x^4(3x - 4)^2$   
e)  $x^2 - 2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2$   
f)  $75a^6b^2c^2 + 108a^2b^8c^2 + 180a^4b^5c^2 = 3a^2b^2c^2(25a^4 + 36b^6 + 60a^2b^3)$   
 $= 3a^2b^2c^2(5a^2 + 6b^3)^2$   
g)  $\frac{9}{4}x^4y - x^3y^2 + \frac{x^2y^3}{9} = \frac{1}{36}x^2y(81x^2 - 36xy + 4y^2) = \frac{1}{36}x^2y(9x - 2y)^2$   
h)  $(2a - 1)^2 - 4(2a - 1) + 4 = [(2a - 1) - 2]^2 = (2a - 3)^2$   
i)  $0,04a^2 + 0,04ab + 0,01b^2 = (0,2a + 0,1b)^2$

4) Ontbinden van viertermen: derdemacht van een tweeterm:

a)  $a^3 + 3a^2 + 3a + 1 = (a + 1)^3$

b)  $8x^3 + 12x^2y + 6xy^2 + y^3 = (2x + y)^3$

c)  $27a^3x^6 - 54a^2bx^4 + 36ab^2x^2 - 8b^3 = (3ax^2 - 2b)^3$

d)  $\frac{1}{27}a^6 - a^5 + 9a^4 - 27a^3 = \frac{1}{27}a^3(a^3 - 27a^2 + 243a - 729) = \frac{1}{27}a^3(a - 9)^3$

5) Ontbinden van viertermen: termen groeperen, 2 en 2:

a)  $ax + bx + ay + by = (a + b)x + (a + b)y = (a + b)(x + y)$

b)  $a - ab + 3 - 3b = a(1 - b) + 3(1 - b) = (1 - b)(a + 3)$

c)  $x^4 - 2x^3 + 4x - 8 = x^3(x - 2) + 4(x - 2) = (x - 2)(x^3 + 4)$

d)  $a^2y - b^2 + b^2y - a^2 = (a^2 + b^2)y - (a^2 + b^2) = (a^2 + b^2)(y - 1)$

e)  $a^3 + a^2b + ab^2 + b^3 = a^2(a + b) + b^2(a + b) = (a^2 + b^2)(a + b)$

f)  $6a^3 + a^2 - 18a - 3 = a^2(6a + 1) - 3(6a + 1) = (6a + 1)(a^2 - 3)$

g)  $15x^2 - 30xy + 8xz - 16yz = 15x(x - 2y) + 8z(x - 2y) = (x - 2y)(15x + 8z)$

h)  $(x + y)^2 + (x - y)(x + y) = (x + y)(x + y) + (x - y)(x + y)$   
 $= (x + y)(x + y + x - y) = (x + y)2x = 2x(x + y)$

i)  $(a - 2b)(a - 4b) - (a - 2b)^2 = (a - 2b)(a - 4b) - (a - 2b)(a - 2b)$   
 $= (a - 2b)[a - 4b - (a - 2b)] = (a - 2b)(a - 4b - a + 2b)$   
 $= (a - 2b)(-2b) = -2b(a - 2b)$

6) Ontbinden van viertermen: termen groeperen, 3 en 1:

a)  $x^2 + 2x + 1 - y^2 = (x + 1)^2 - y^2 = [(x + 1) + y][(x + 1) - y] = (x + 1 + y)(x + 1 - y)$

b)  $a^2 - c^2 + 4ab + 4b^2 = (a + 2b)^2 - c^2 = (a + 2b + c)(a + 2b - c)$

c)  $16a^2 - 9b^2 - 4c^2 + 12bc = 16a^2 - (9b^2 + 4c^2 - 12bc) = 16a^2 - (3b - 2c)^2$   
 $= [4a + (3b - 2c)][4a - (3b - 2c)]$   
 $= (4a + 3b - 2c)(4a - 3b + 2c)$

d)  $x^4 - 9b^2x^2 - 6bc^2x - c^4 = x^4 - (3bx + c^2)^2 = (x^2 + 3bx + c^2)(x^2 - 3bx - c^2)$

e)  $25a^2 - 20ab + 4b^2 - 9a^4 = (5a - 2b)^2 - 9a^4 = (5a - 2b + 3a^2)(5a - 2b - 3a^2)$

f)  $c^2 - d^2 - 2c + 1 = (c - 1)^2 - d^2 = (c - 1 + d)(c - 1 - d)$

g)  $4 - a^2 - 2ax - x^2 = 4 - (a^2 + 2ax + x^2) = 4 - (a + x)^2 = (2 + a + x)(2 - a - x)$

7) Ontbinding in factoren: allerlei:

a)  $a^2 - ab = a(a - b)$

b)  $a^2 - 4y^4 + 12x^3y^2 - 9x^6 = a^2 - (4y^4 - 12x^3y^2 + 9x^6)$   
 $= a^2 - (2y^2 - 3x^3)^2$   
 $= (a + 2y^2 - 3x^3)(a - 2y^2 + 3x^3)$

c)  $32x^4 - 2a^4 = 2(16x^4 - a^4) = 2(4x^2 + a^2)(4x^2 - a^2)$   
 $= 2(4x^2 + a^2)(2x + a)(2x - a)$

d)  $27x^3 - 27x^2y + 9xy^2 - y^3 = (3x - y)^3$

e)  $36a^4b^3c - 24a^2b^2c + 4bc = 4bc(9a^4b^2 - 6a^2b + 1)$   
 $= 4bc(3a^2b - 1)^2$

f)  $27a^5 + a^2 - 108a^3 - 4 = a^2(27a^3 + 1) - 4(27a^3 + 1)$   
 $= (27a^3 + 1)(a^2 - 4)$   
 $= (27a^3 + 1)(a + 2)(a - 2)$

g)  $25a^6b^2 - 10a^5b^3 = 5a^5b^2(5a - 2b)$

h)  $125a^3 - 75\frac{a^2}{b} + 15\frac{a}{b^2} - \frac{1}{b^3} = \left(5a - \frac{1}{b}\right)^3$

i)  $(a+b)^2 + 2(a+b) + 1 = [(a+b) + 1]^2$   
 $= (a + b + 1)^2$

j)  $20a^3 + 5a^2 - 4a - 1 = 5a^2(4a + 1) - (4a + 1)$   
 $= (4a + 1)(5a^2 - 1)$

k)  $a^2 - 6a + 9 - b^2 = (a - 3)^2 - b^2$   
 $= (a - 3 + b)(a - 3 - b)$

l)  $\frac{1}{9}a^6b^4 - \frac{2}{9}a^5b^2 + \frac{1}{9}a^4 = \frac{1}{9}a^4(a^2b^4 - 2ab^2 + 1)$   
 $= \frac{1}{9}a^4(ab^2 - 1)^2$

m)  $x(a-1) - y(1-a) = x(a - 1) + y(a - 1)$   
 $= (a - 1)(x + y)$

n)  $a^2 - 2ab + 2bc - ac = a(a - 2b) + c(2b - a)$   
 $= a(a - 2b) - c(a - 2b)$   
 $= (a - 2b)(a - c)$

o)  $x^{16} - 1 = (x^8 + 1)(x^8 - 1) = (x^8 + 1)(x^4 + 1)(x^4 - 1)$   
 $= (x^8 + 1)(x^4 + 1)(x^2 + 1)(x^2 - 1)$   
 $= (x^8 + 1)(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

p)  $8a^5 + 24a^3b^4 + 18ab^8 = 2a(4a^4 + 12a^2b^4 + 9b^8)$   
 $= 2a(2a^2 + 3b^4)^2$

q)  $64x^6 - 1 = (8x^3 + 1)(8x^3 - 1)$

r)  $150ab^2 + 8a^3 - 60a^2b - 125b^3 = (2a - 5b)^3$

s)  $a^3 - b^4 + a^2b - ab^3 = a^2(a + b) - b^3(a + b)$   
 $= (a + b)(a^2 - b^3)$

t)  $4x^4 - 81 = (2x^2 + 9)(2x^2 - 9)$

u)  $3a^4b + 24ab^4 = 3ab(a^3 + 8b^3)$